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INTERPLANETARY GAS. XIII. GROSS PLASMA VELOCITIES FROM THE ORIENTATIONS OF IONIC COMET TALES

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INTERPLANETARY GAS. XIII. GROSS PLASMA VELOCITIES FROM THE ORIENTATIONS OF IONIC COMET TAILS*

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ABSTRACT

Results are presented concerning the velocity of the solar-wind plasma as deduced from the observations of Type I (ion) tails contained in the comet catalogue of Belton and Brandt. The analysis is in the spirit of the early work by Biermann, where the orientations are interpreted on the basis of dynamical aberration. The major conclusions are as follows: (i) The solar wind has a fairly sharp velocity minimum at 150 ± 50 km/sec; this agrees with the prediction of Axford, Dessler, and Gottlieb. (ii) The solar-wind velocity is correlated with geomagnetic activity in a manner which confirms the relationship found from the Mariner II data by Snyder, Neugebauer, and Rao and from the travel times for geomagnetic storms by Hirshberg. The comet tail, Mariner II, and storm data are consistent with the expressions

$$w = 330 + 9.0\ \Sigma K_p\ (\text{km/sec}) \quad \text{or} \quad w = 316\ A_p^{0.20}\ (\text{km/sec})\,,$$

where w is the solar-wind velocity, ΣK_p is the sum of K_p for the day in question, and A_p is the surface magnetic disturbance index. These statistical relations hold over nearly the entire range of the magnetic indices. (iii) These relations predict a mean plasma velocity of 500 km/sec with only a 50 km/sec spread in the means between years of solar maximum and solar minimum.

(iv) Evidence is found for a mean tangential component of the solar-wind velocity near Earth of ≈ 10 km/sec directed in the sense of the solar rotation; the value of the tangential velocity is apparently lower for quiet conditions and higher for disturbed conditions in the interplanetary medium. (v) The sample of Type I comet tails (both direct and retrograde) is not uniformly distributed in space and time, and care must be exercised in drawing conclusions from the sample presently available; such bias may (at least in part) account for the curious dependence of the solar-wind velocity on heliographic latitude found by Pflug, where the minimum velocity is found in the sunspot zone. It appears that the latitude dependence of the plasma velocity cannot be determined with the information presently available.

I. INTRODUCTION

a) History

The physics of comets and particularly their tails is now known to be closely linked with the properties of the interplanetary gas. Many cometary phenomena can be regarded as the result of the interaction between the solar wind and the cometary material. Over a decade ago, Biermann (1951, 1953) used Hoffmeister's (1943) observations of orientations of ion tails to confirm the approximate value of the plasma velocity derived from the study of tail knots; the basic assumption is that the average orientation is determined by dynamical aberration, i.e., by the direction of motion of the solar-wind plasma as seen by an observer on the comet.

Brandt (1961) suggested that a model of the interplanetary plasma could be constructed from a large sample of observations of comet-tail orientations; the first attempt at such a sample is now available in the catalogue of Belton and Brandt (1966). These authors have emphasized that the primary value of the catalogue is in the basic data and no interpretation is found therein. The present paper deals exclusively with the interpre-

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tation of the Type I or ion tails; Belton (1965*a*, *b*) has considered the interpretation of the Type II or dust-tail data contained in the catalogue.

After the Introduction, the sections of this paper deal, in turn, with the minimum plasma velocity, the correlation of the plasma velocity with geomagnetic activity, the tangential or azimuthal velocity of the plasma, and the possible ramifications of a bias in the sample. The remainder of the Introduction deals with the form of the data and the equations needed.

b) Basic Relations and Concepts

The comet catalogue (Belton and Brandt 1966) gives the aberration angle ϵ which is the angle between the axis of the comet tail and the prolonged radius vector; this angle follows from the assumption that the tail lies in the plane of the orbit of the comet, the geometry, and the measurement or computation of the position angle (θ) of the tail, the position angle (ϕ) of the prolonged radius vector, and the position angle (ψ) of the velocity vector backward along the orbit of the comet. Also tabulated are various quantities describing the position and velocity of the comet.

The velocity of the solar wind then follows from the assumption of dynamical aberration, and the equation is

$$\tan\epsilon = \frac{V \sin \gamma - w_t \cos i}{w_r - V \cos \gamma}, \tag{1}$$

where ϵ is the aberration angle (described above), V is the velocity of the comet, γ is the angle between the radius vector and the comet's velocity vector V, w_r is the radial velocity of the solar plasma, w_t is the tangential velocity (taken positive in the sense of solar rotation), i is the inclination of the comet's orbit to the plane of the ecliptic, and $w_t \cos i$ is approximately the component of the solar-wind velocity in the plane of the comet's orbit and perpendicular to the radius vector $\boldsymbol{r}$. Note that the existence of a tangential component generally implies that the tail vector $\boldsymbol{t}$ cannot lie exactly in the plane of the orbit. Mammano and Wurm (1965) have studied Comet Daniel (1907D) in an attempt to study departures of an ionic tail from the orbital plane because the observations were obtained when Earth was essentially in the orbital plane of the comet. Mammano and Wurm (1965) find that *on the average* the tail of Comet Daniel was essentially in the orbital plane of the comet; this result is not too surprising because of Comet Daniel's low inclination of 7°.

In much of the following discussion, the tangential velocity is taken as zero. The solar-wind velocity thus defined is tabulated by Belton and Brandt (1966). In subsequent development, w stands for the radial velocity of the solar wind.

This investigation is limited to approximately 600 photographic observations of tails considered to be Type I. The individual observations are usually fairly uncertain, and the discussion is usually in terms of groups of observations. Sources of uncertainty are: (1) real variations within the group; (2) possible effects of a tangential motion of the plasma if not specifically taken into account (see § IV); (3) the possibility that the tail does not lie in the comet's orbital plane; (4) the difficulty of measuring the correct tail axis toward which the tail streamers turn (see Mammano and Wurm 1965); (5) measuring errors (estimated to be $1^\circ\!.5$ in θ); (6) computational errors; (7) incorrect tail types. The problem is further complicated by the error in w being a function of ϵ. Thus, for an ϵ of 10°, a 1° error is a minor complication, but for an ϵ of 1°, a 1° error is a disaster. The result is that velocities in the range 200–300 km/sec and below are comparatively well determined while larger values *individually* contain usually only the information that the velocity is large compared to 200–300 km/sec. Indeed, some perfectly valid observations are displaced by errors into the "forbidden quadrant" preceding the comet with ϵ negative. Any acceptable scheme of averaging must take full account of these observations.

The procedure adopted here (and by others) is to form means of w^{-1}. As a working variable, w^{-1} may be positive, zero, or negative and thus all appropriate observations are included. The crucial point is the behavior of the working variable near $\epsilon = 0$, where w^{-1} varies smoothly and has no singularities. This criterion is *not* met by w, which tends to $+\infty$ or $-\infty$ depending on which way $\epsilon = 0$ is approached. Physically, one may think of $1/w$ as being directly proportional to tan ϵ (see eq. [1]). However, the adopted procedure introduces another problem, namely, that $\langle w \rangle$ is generally not equal to $\langle 1/w \rangle^{-1}$, and usually $\langle w \rangle > \langle 1/w \rangle^{-1}$, by Cauchy's inequality. An attempt to treat this situation is contained in an earlier report (Brandt 1965, Table I). There, the difference between the two means was estimated using a distribution function of the form, $f(w) = \exp - [(w - w_0)/\delta w]^2$. However, the preliminary treatment neglected two points which *tend* to preserve equality between $\langle w \rangle$ and $\langle 1/w \rangle^{-1}$. (1) The crucial parameter in the comparison is the dispersion δw which must be computed from the spread in the values of $1/w$ or a quantity $\delta(1/w)$. Now a given value of $\delta(1/w)$ or the dispersion in $1/w$ implies a sharper cutoff in $f(w)$ on the low-velocity side of the mean than on the high-velocity side. Thus, the use of a mean δw results in an underestimate of $\langle 1/w \rangle^{-1}$. (2) The use of an idealized form for $f(w)$ also entirely neglects the effects of the negative values of $1/w$ (which correspond to very high velocities) which are used in forming the means. The result again is to underestimate the value of $\langle 1/w \rangle^{-1}$. Since the two effects cited tend to restore equality of $\langle w \rangle$ and $\langle 1/w \rangle^{-1}$, and in view of our ignorance concerning the distribution function in w or $1/w$, we adopt the values $\langle 1/w \rangle^{-1}$ without further correction as being representative of $\langle w \rangle$.

The geometrical configuration appropriate to the observations is variable and strongly influences the accuracy of the determination of ϵ. The accuracy must be poor, for example, when Earth is near the plane of the orbit of the comet under observation. Belton and Brandt (1966) have tabulated the quantity $d\epsilon/d\theta$, which gives the rate of change of the aberration angle ϵ with respect to the measured position angle of the tail, θ. Thus, we may recall that θ has an average uncertainty of approximately 1°5 to compute an uncertainty in ϵ arising from errors of measurement. As one can see from equation (1), $1/w$ is proportional to tan ϵ (since $w \gg V \cos \gamma$), and hence, the uncertainty in $1/w$ is proportional to $\sec^2 \epsilon \, (d\epsilon/d\theta)$. We then define a relative weight by

$$\mathfrak{W} = \left(\sec^2 \epsilon \frac{d\epsilon}{d\theta}\right)^{-2}, \tag{2}$$

and group means are formed by

$$\langle 1/w \rangle = \frac{\sum_i (1/w)_i \mathfrak{W}_i}{\sum_i \mathfrak{W}_i}. \tag{3}$$

The probable error of the mean can then (optimistically) be computed from

$$\mu = (0.6745) \left[\frac{\sum_i [(1/w)_i - \langle 1/w \rangle]^2 \mathfrak{W}_i}{(n-1) \sum_i \mathfrak{W}_i}\right]^{1/2}, \tag{4}$$

where n is the number of independent observations in the group. In the discussion, we refer to the quantity defined by equation (4) but without the factor of $(n - 1)^{1/2}$ in the denominator as the "intrinsic dispersion."

The Appendix gives a tabulation of the comets used, the number of observations of each, and the total weight per comet. Notice that the distribution of weights among comets is very uneven with a small number of comets comprising the bulk of the total weight; for example, Comets Morehouse (1908C), Whipple-Fedtké-Tevzadze (1942G), and Mrkos (1957D) comprise about three-fourths of the total weight of observations used in this investigation. This undesirable but unavoidable situation is further discussed in §§ V and VI.

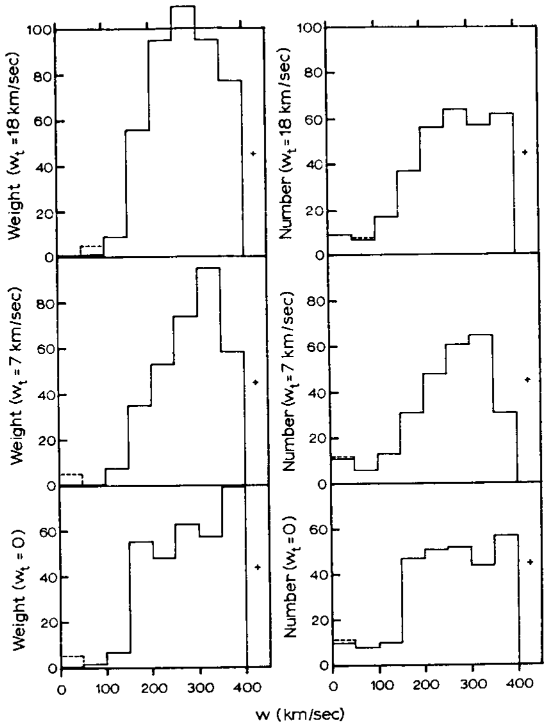

FIG. 1.—Histograms showing the lower cutoff in the solar-wind velocity. The plots are by weight and number and for a tangential velocity of 0, 7, and 18 km/sec. The plus sign to the right of each diagram symbolically represents the additional observations with velocities greater than 400 km/sec which are not shown. The dotted part of the histograms between 0 and 100 km/sec shows the result of the inclusion of a suspect observation; see text for further discussion.

II. THE MINIMUM VELOCITY OF THE SOLAR WIND

Here, we are interested in the velocities below 200–300 km/sec and hence (as explained above) we may use the individual velocities as tabulated in the catalogue. These are grouped by 50 km/sec intervals and plotted in a histogram by number and by weight. Figure 1 shows these results for assumed tangential velocities (w_t) of 0, 7, and 18 km/sec. As is found in § IV, the most probable value of the tangential velocity for the "quiet conditions" corresponding to these low velocities is between 0 and 7 km/sec.

The histograms in Figure 1 clearly show the cutoff in the solar-wind velocity at 150 km/sec with an estimated error of ± 50 km/sec. The reality of the cutoff is strongly reinforced by the fact that it is sharper by weight than by number; thus, the handful of observations below the cutoff are preferentially of low quality.

This point can be illustrated with the following figures which apply to the velocities below 100 km/sec. The total weight of the twenty observations below 100 km/sec is about 6, 5 of which are contained in a single observation (No. 92). A check of this observation discloses that it is considered very uncertain by the observer (Curtiss 1903). If this observation is omitted, the average weight below 100 km/sec is ≈ 0.05 as compared to an average of ≈ 1.4 for all 604 photographic observations of Type I comet tails. In Figure 1 the dotted portion of the diagram between 0 and 100 km/sec shows the situation with the inclusion of the suspect observation, and the corresponding solid line represents the results with it removed.

This lower bound is of interest in connection with the origin of the solar wind and, in particular, with the prediction of Axford, Dessler, and Gottlieb (1963) that the solar-wind velocity is either zero or greater than about 100 km/sec. Their argument is based on the fact that the solar magnetic field would confine the plasma unless the velocity were sufficiently high; the values of the cutoff velocity found here (150 km/sec) and predicted (100 km/sec) should be regarded as in essential agreement because of the uncertainty of various quantities entering into the calculations.

III. THE CORRELATION OF THE SOLAR-WIND VELOCITY WITH GEOMAGNETIC ACTIVITY

For some years geophysicists and astronomers have been interested in determining the paths of particles ejected from the Sun. The general problem is of interest in auroral theories, and received added impetus with Bartel's (1932) postulation of the solar "M regions" thought to be responsible for the weaker geomagnetic disturbances. It is now over 30 years later and the solar M regions are still not identified; indeed, their association with active regions is still controversial. Mustel (1964, 1965) affirms that active regions are responsible while Allen (1944), Pecker and Roberts (1955), and Saemundsson (1962) deny this. Even the Mariner II data with its relatively precise velocity determinations have not led to the resolution of the dilemma as reported by Snyder, Neugebauer, and Rao (1963). Recently, Saito (1965) has reported evidence of non-radial plasma ejection in connection with the recurrent magnetic storms. The situation is undoubtedly complex and is made more so by the interaction between the components of the solar plasma traveling at different velocities and the possibility of motion in latitude. Thus, it is clear that our knowledge of the trajectories of particles ejected from the Sun is quite incomplete. It is rather difficult to believe on physical grounds that the M regions are associated with anything but the active solar regions or "centers of activity." The significant point of interest here is that it has not been possible to establish this result incontrovertibly on the basis of trajectories even with the velocities known (Mariner II).

The problem of the analysis of the comet cata is to compute a "time differential" to be applied to the time of comet observation; this "time differential" refers the time to Earth and one can then utilize the tabulated geomagnetic indices. Our primary interest is in the long-lived streams or beams which show the 27-day recurrence; the problem is rendered tractable by the relatively large width or thickness over which the properties can be regarded as essentially constant. Brandt and Cassinelli (1966) find a beam width of 43° or 3.2 days from the Mariner II data while Mustel (1964) gives 4.5 days.

We proceed under the assumption that the plasma is emitted radially with a width of about 40° having a velocity which is constant with time. Even though all particles travel radially, the beam is shaped like an Archimedes spiral (Parker 1958) because of the solar rotation. The time delay is easily shown to be

$$T = \frac{r_E - r_c}{w} + \frac{l_E - l_c}{\Omega} \tag{5}$$

where the subscripts E and c refer to Earth and the comet, respectively; r is the heliocentric distance, l is the heliocentric longitude, and Ω is the synodic rate of solar rotation ($\approx 13^\circ.3$ per day); this equation has been given by Snyder *et al.* (1963), and the interpretation of the second term is obvious. The first term has been called a "radial time lag" by Snyder *et al.*, but this terminology (while entirely accurate for the case of a spherically symmetric blast wave) may be misleading for the case of a beam with constant properties corotating with the Sun. Generally, the same plasma never encounters both Earth and the comet. Because we have assumed that nothing varies along the beam, there is no value in "labeling" a particular clump of plasma; all that matters is the associated time delay for encountering the beam. Thus, the first term could be interpreted as a "curvature term" caused by the form of the Archimedes spiral. The geometrical interpretation leads to an identical expression. Note, however, that the "curvature term" *implicitly* depends on the solar rotation period and would vanish for a non-rotating Sun.

Consider now the two terms in equation (5). The second term depends only on the position of the comet and is straightforward to apply. The first term, however, depends on the velocity associated with the individual observations which is very poorly known, and which sometimes has no physical significance (for the case of ϵ negative). The first term is generally small because comets are preferentially observed near (in heliocentric distance) Earth. The most distant observations from Earth which are crucial to the relation found between plasma velocity and geomagnetic activity are those of Comet Whipple-Fedtké-Tevzadze (1942G). The observations are taken while the comet was at a heliocentric distance of about 1.5 a.u. and gave typical velocities of 750 km/sec. These figures give curvature times of about 1.1 days, a figure which should be compared to 3.2 days or 4.5 days for the typical beam widths near Earth according to the figures quoted above. Essentially the same result holds for lower velocities because they are preferentially observed nearer Earth. The pertinent comets are Mrkos (1957D) and Finsler (1937F), which have *mean* differences of the heliocentric distances of Earth and the comet of 0.3 a.u. and 0.1 a.u., respectively. Thus, the curvature time is small compared to the width of the beam in our sample of observations. Moreover, Biermann (1953) has identified activity in the tail knots of Comet 1942G on March 29/30, 1943, with the geomagnetic activity *on the same dates*. The daily sum (ΣK_p) of the K_p for these dates averages 31, which corresponds (see eq. [8]) to a mean velocity of 600 km/sec. These observations could indicate a beam of less curvature (or more nearly radial) than does equation (5).

Finally, the nature of the distribution (Bartels 1963, Fig. 5) of the geomagnetic K_p indices is crucial. Mean characteristics are shown in Table 2 (see below). The mean is thus basically quiet with $K_p \approx 2$, and the higher K_p values (typically 4 to 5 for an M region) are statistically rare. It is clear that the higher K_p values *must* be correctly included in attempting to find a relation between K_p and w. The higher K_p values should naturally correspond to the more nearly radial beams. *Thus, if one assumes a radial beam and finds a high K_p, the use of the radial beam was correct. If, however, a low K_p is found, the result is still correct on the average.* In other words, it is preferable to err in the direction of the assumed beams being too radial in order to correctly include the relatively rare values of high K_p.

In view of the arguments just presented, the observed times are corrected only by the longitude difference in the discussion to follow. This simplifying assumption is unquestionably a weak point in the analysis presented here; attention must be paid to the particle trajectories in the future. However, some detailed knowledge concerning the interaction of the fast and slow beams seems to be a prerequisite. By way of comparison, the total time delays, T, for the Mariner II data were mainly small but showed substantial periods with values of $\frac{1}{2}$ to 1 day; the longitude correction was generally the larger when T was significant. A total time delay of zero was assumed in deriving the correlations and relations reported by Snyder *et al.* (1963), a procedure which resulted in virtually no loss of accuracy partially because of the wide beam width and partially because of the fortuitous geometry.

The indices K_p are available from 1932 (Bartels 1962), a fact which reduces the total number of observations at all latitudes to about 270; in subsequent investigations, it may be possible to extend the analysis farther back in time by means of the various interrelations between the different geomagnetic indices. No attempt has been made to distinguish streams from transient outbursts. The longitudinal time delays were computed and the corresponding ΣK_p tabulated. The data were divided into four groups by ΣK_p and the weighted means of ΣK_p and $(1/w)^{-1}$ formed by latitude groups; the results are shown in Figure 2. Note that we are not seeking a latitude variation of the solar-wind velocity. The purpose of the latitude groups is simply to see if comets at high latitude might not be correlated with geomagnetic activity because of a finite extent of the beam in latitude. It does not appear possible to establish (at present) a variation of the plasma velocity with latitude (see § V).

The uncertainties are appreciable as shown by the error bars, but the general trend as shown by the points and by the least-squares lines derived from them is quite similar for all of the latitude groups. Fortunately, comets are usually observed at low latitudes so that the question of the extent of solar M regions *in latitude* is not crucial. The available evidence (Antrack, Biermann, and Lüst 1964; Biermann and Lüst 1964) indicates that storms extend at least to 45° latitude. Lüst (1961) found evidence for a storm-related correlation when Earth and Comet Swift (1899I) had a latitude difference

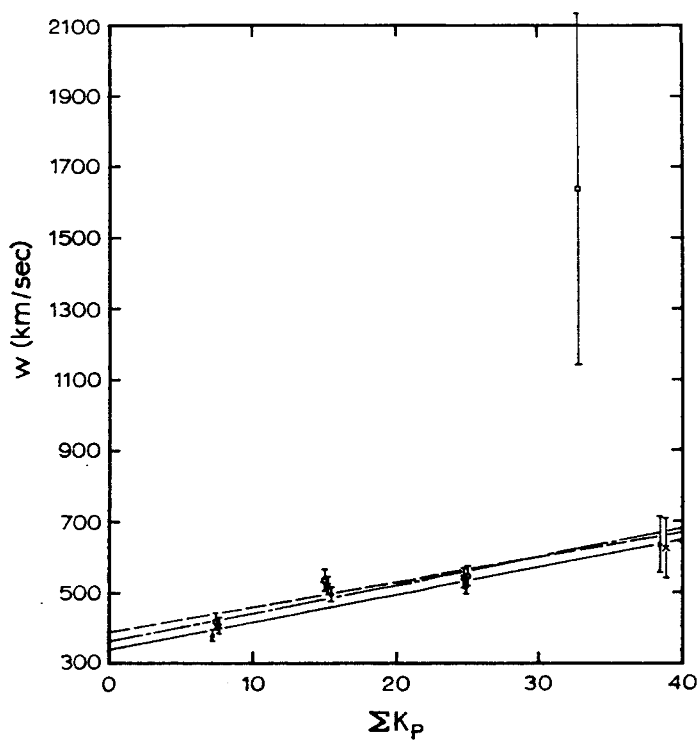

FIG. 2.—The various solutions for the w versus ΣK_p relation based solely on cometary data. The solution with all observations is denoted by filled circles and the least-squares fit, $w = 337 + 7.78\ \Sigma K_p$, is shown by a solid line. The solution restricted to solar latitudes less than $\pm 70°$ is rather similar to the total solution, and the least-squares fit, $w = 333 + 8.11\ \Sigma K_p$, is not shown. The solution restricted to solar latitudes less than $\pm 45°$ is denoted by crosses, and the least-squares fit, $w = 363 + 7.98\ \Sigma N_p$, is shown as a dot-dashed line. Finally, a solution restricted to solar latitudes less than $\pm 25°$ is denoted by squares, and the least-squares fit, $w = 391 + 6.99\ \Sigma K_p$, is shown as a dashed line. The error bars give the probable error of the mean. The errors shown are appreciable, but the general trend in all cases is quite similar. Note that no latitude dependence of solar-wind velocity is implied here.

of 39°. Since the extent of M regions in longitude is approximately 45°, the same extent would be expected in latitude unless the lower plasma densities believed to prevail at high latitudes permit a larger extent in latitude due to expansion. In any event, there appears to be a good chance of a significant correlation if the disturbed region, Earth, and the comet are in the same hemisphere. Hence, the basic agreement shown in Figure 2 may not be too surprising.

Note, however, that certain geomagnetic storm data (Priester and Cattani 1962; Mustel 1964) and the occurrence of aurora (Chamberlain 1961) could be interpreted in terms of a small extent in latitude for the beams. However, examination of this evidence shows that it supports a small beam width only on the side of the beam toward the solar equator. Geomagnetic evidence cannot be used to investigate the extent of the beams from the active regions towards the polar regions. Indeed, the geomagnetic and comet data, taken together, argue for a small extent of the beam in latitude from the active regions or the sunspot belt toward the equator and a large extent toward the polar regions.

The solution for all latitudes is adopted because it is basically similar to the others and because the random and systematic errors are reduced due to the larger number of observations and independent comets. The details are given in Table 1. The entries are

TABLE 1

CORRELATION OF COMET-DEDUCED VELOCITIES WITH THE DAILY SUM OF K_p

Group	ΣK_p Range	No. of Observa-tions	$\langle \Sigma K_p \rangle$	$\langle 1/w \rangle^{-1}$ (km/sec)	Intrinsic Dispersion, $\delta(1/w)$ (sec/km)	Probable Error of $\langle 1/w \rangle^{-1}$ (km/sec)
1..........	0–10	67	7	380	0.00090	16
2..........	10–20	86	15	495	.00075	20
3..........	20–30	70	25	515	.00061	19
4..........	30 and above	47	39	635	0.00130	78

generally self-explanatory; the intrinsic dispersions in $(1/w)$ are converted into a probable error of $\langle 1/w \rangle^{-1}$ via the relation $|w^2\, \delta(1/w)| = \delta w$ and by introducing the factor $\sqrt{(n-1)}$ in equation (4).

The four adopted mean points (with probable errors) are plotted in Figure 3 along with the points from Mariner II (Snyder *et al.* 1963) and the points from the travel times of geomagnetic storms as determined by Hirshberg (1965). The method of reduction for the Mariner II data was described above. The storm data (Hirshberg 1965) is based solely on cases where the identification of flare event is certain, and on the average of the 3-hour range indices a_p for the first four 3-hour periods after the storm sudden commencement (SSC). The a_p values given by Hirshberg have been converted to ΣK_p via a statistical interrelation (Bartels 1962).

The uncertainties, as revealed by the error bars in Figures 2 and 3, are appreciable as would be expected from crude nature of the analysis. Nonetheless, the correlation between plasma velocity and geomagnetic activity via ΣK_p is clearly shown. The least-squares line for the comet data has been derived with the points given relative weights which are inversely proportional to the square of the probable error; the equation for the adopted solution is

$$w = 337 + 7.78\ \Sigma K_p (\text{km/sec})\ . \tag{6}$$

This is to be compared with the relation

$$w = 330 + 8.44\ \Sigma K_p\ (\text{km/sec}) \tag{7}$$

found by Snyder *et al.* (1963). The agreement is entirely satisfactory, and at least partly fortuitous.

Now the intrinsic dispersion for Group 4 (in Table 1) would be expected to be somewhat higher than the other groups because of the larger range in ΣK_p and because it preferentially includes the observations with negative ϵ. Hence, Group 4 is not a proper indicator of a possible increase in intrinsic dispersion due to a physically significant grouping. However, the intrinsic dispersions listed in Table 1 for Groups 1, 2, and 3 are about a factor of $\frac{2}{3}$ smaller than intrinsic dispersions obtained as a result of other groupings such as by heliocentric distance, heliocentric latitude, and by sunspot maximum

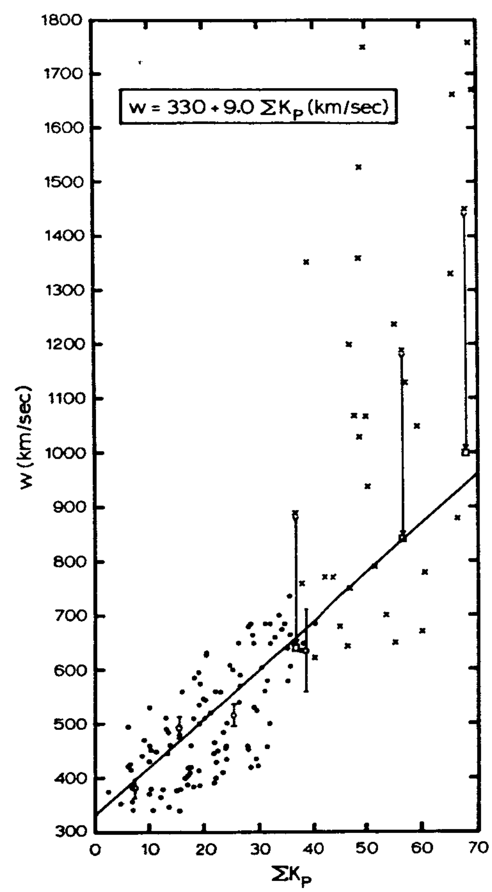

FIG. 3.—The data from comets, Mariner II, and geomagnetic storms plotted together to show a mean w versus ΣK_p relation. The comet data are shown by open circles (with error bars), the Mariner II data are plotted as dots, and the observed storm data as crosses; the squares are nearly on the mean line for the storm data after correction for the detached shock. The arrows show the amount of the correction, and the three original points associated with the arrows lie nearly on the mean line for the uncorrected storm data. The solid mean line was drawn by eye and has the equation shown.

and minimum as shown in Table 3 (see below). This fact definitely supports the reality of the results.

Nevertheless, the primary value of the cometary data does not lie in the detailed numerical results. Rather, the primary value lies in the independent confirmation of the basic relation found by Snyder *et al.* for a wider range of distances and latitudes and throughout the solar cycle.

A mean line taking into account the comet, Mariner II, and storm data has been drawn in Figure 3. It has the equation

$$w = 330 + 9.0\ \Sigma K_p (\mathrm{km/sec})\ . \tag{8}$$

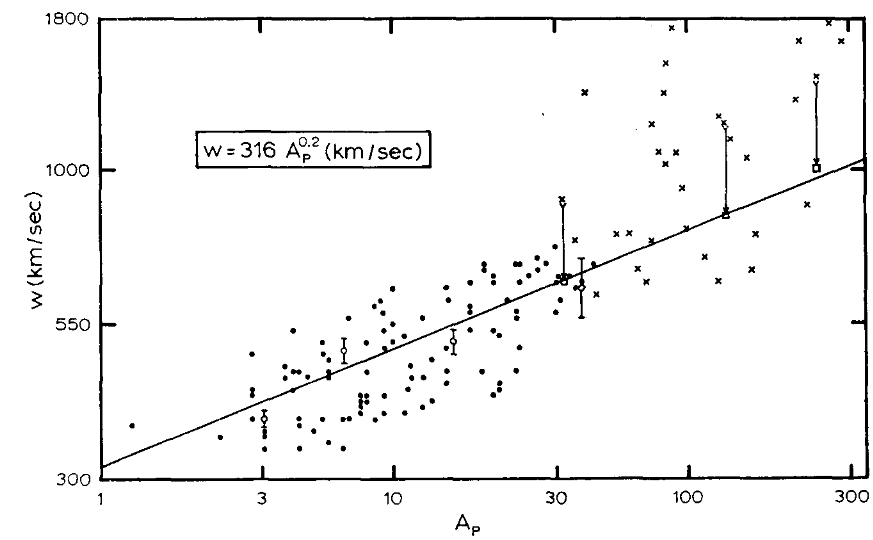

FIG. 4.—The data from comets, Mariner II, and geomagnetic storms plotted together to show a mean w versus A_p (logarithmic) relation; see caption to Fig. 3 for the symbols. The mean line has the equation shown and was drawn by eye.

The data in Figure 3 have been replotted in terms of the daily equivalent planetary amplitude, A_p, in Figure 4 (see Maer and Dessler 1964). Again, a mean line for all the data has been drawn. It can be represented by

$$w = 316\ A_p^{0.20} (\mathrm{km/sec}). \tag{9}$$

Note that Hirshberg (1965) considers the directly determined velocities for storms too high by some hundreds of km/sec because the sudden storm commencement is probably produced by a shock which proceeds the flare-associated plasma. The picture presented is entirely reasonable, and the nature of the correction is indicated in Figures 3 and 4.

Thus, we have the following situation. With the various provisos and simplifications kept in mind, we have in equations (8) and (9) a statistical relationship between solar plasma velocity and magnetic disturbance which is valid over essentially the entire range of geomagnetic activity.

Let us now utilize these mean relations to investigate the variation of the *mean* plasma velocity throughout the 11.2 year solar cycle. Bartels (1963) has classified the years in the solar cycle into five groups and has tabulated the frequency of occurrence of the different values of K_p. Mean K_p's have been computed from Bartels' figures (1963, Fig.

5) and these are readily converted into mean velocities for the class years in question using equation (8). The results are shown in Table 2; here $\langle R \rangle$ is the mean Zürich relative sunspot number.

The *mean* plasma velocity is conspicuous by its lack of variation through the solar cycle. Snyder *et al.* (1963) found no dependence of the plasma velocity on the sunspot number. Thus, the yearly mean velocity of the solar wind is essentially constant throughout the solar cycle; the years of maximum and minimum differ basically only in the relative frequency of the more intense and rare geomagnetic disturbances and associated high plasma velocities.

IV. POSSIBLE TANGENTIAL VELOCITY OF THE SOLAR PLASMA

The value of the tangential velocity is of interest in the theory of the origin of the solar wind and, in particular, in terms of the interaction between the solar plasma and the extended solar (now interplanetary) magnetic field. The study of comet tails can be of value in this problem because a tangential component of the solar-wind plasma should produce a systematic shift in the tail orientations of direct comets relative to retrograde comets. For example, if the comet's velocity is in the same direction as the tangential velocity of the plasma, the azimuthal velocity of the comet relative to the plasma is much reduced.

TABLE 2

MEAN PLASMA VELOCITY AND THE SOLAR CYCLE

Bartels' Class	Description	$\langle R \rangle$	$\langle K_p \rangle$	$\langle w \rangle$ (km/sec)
A.	Minimum	6.4	1.8_5	463
B.	Ascending	61.5	1.9_9	473
C.	Maximum	152.0	2.5_1	511
D.	After maximum	119.6	2.5_4	513
E.	Before minimum	34.2	2.5_4	513
. . . .	All	75.3	2.3_5	499
. . . .	Typical M region	. . . .	4.2_9	639

Consider now equation (1) with the term in $w_t \cos i$ retained. The angle i is the inclination of the orbit of the comet to the plane of the ecliptic; it is 0°–90° for direct comets and 90°–180° for retrograde comets. If one assumes that the tangential velocity is related to the solar rotation, one should use the plane of the solar equator instead of the plane of the ecliptic. These planes differ by only 7° inclination, and, hence, no distinction is made in this investigation. Equation (1) can be rewritten for direct and retrograde comets by taking absolute values of $\cos i$ and explicitly exhibiting the sign. If w_r and w_t are assumed to be constant, the difference of the equations for direct and retrograde comets gives

$$w_t \cong \frac{[\langle \tan \epsilon \rangle_R - \langle \tan \epsilon \rangle_D]}{[\langle |\cos i| \rangle_R + \langle |\cos i| \rangle_D]} \left\langle \frac{1}{w} \right\rangle^{-1} (\mathrm{km/sec}). \tag{10}$$

The quantities needed to evaluate equation (10) are tabulated in Table 3. The complete sample of comets gives a fairly well-determined value of +9 km/sec, i.e., in the direction of the solar rotation. A value ≈ 10 km/sec is probably representative.

The consistency and effects of excluding various comets from the solution can be investigated by dividing the observations into groups of equal weight by heliocentric distance; this is a simple way to separate the individual comets and should not be considered as a search for the heliocentric variation of plasma velocity (see § V). The solution for all observations is the one quoted above (also shown in the last line of Table 3) and is $w_t = 9$ km/sec and $w_r = 448$ km/sec.

When the solutions are split into 2, 3, and 4 groups, the result is that the velocities for the group exterior to 1.4 a.u. (and hence including largely Comet 1942G) increase somewhat with respect to the values quoted in the last paragraph while the solutions for the other groups (and hence, essentially all the other comets) give consistently $w_t \approx 4 \pm 2$ km/sec (p.e.) and $w_r \approx 380$ km/sec. The tangential velocity shows up in *all* the groups and hence appears to be quite real. Now we must ask whether or not this group (or subgroup) is representative of typical interplanetary conditions. Only values

TABLE 3

SOME VELOCITIES INFERRED FROM OBSERVATIONS UNDER VARIOUS ASSUMPTIONS*

r Range (a.u.)	N	$\langle \tan \epsilon \rangle$	Probable Error $\langle \tan \epsilon \rangle$	$\langle 1/w \rangle^{-1}$ (km/sec)	$\langle \cos i \rangle$	Comet Direction	Assumed w_t (km/sec)	Intrinsic Dispersion (sec/km)
<1.4......	143	0.091	0.0040	426	+0.73	Direct	0	0.00110
<1.4......	141	.091	.0040	379	+ .73	Direct	6	.00116
<1.4......	138	.092	.0039	297	+ .73	Direct	15	.00121
<1.4......	310	.104	.0029	361	− .43	Retrograde	0	.00127
<1.4......	310	.104	.0029	386	− .43	Retrograde	6	.00116
<1.4......	311	.104	.0029	419	− .43	Retrograde	15	.00105
<1.4......	453	.100	.0024	378	....	Combined	0	.00122
<1.4......	451	.100	.0024	384	....	Combined	6	.00116
<1.4......	449	.100	.0023	373	....	Combined	15	.00114
All........	214	.064	.0026	571	+ .83	Direct	0	.00094
All........	212	.063	.0025	494	+ .83	Direct	6	.00102
All........	208	.064	.0025	369	+ .83	Direct	15	.00119
All........	375	.090	.0026	390	− .51	Retrograde	0	.00137
All........	375	.090	.0026	425	− .51	Retrograde	6	.00122
All........	377	.092	.0027	437	−0.52	Retrograde	15	.00154
All........	589	.077	.0019	458	....	Combined	0	.00122
All........	587	.077	.0019	455	....	Combined	6	.00114
All........	585	0.079	0.0019	402	....	Combined	15	0.00140
All†.....	585	0.077	0.0019	448	....	Combined	9	0.00112

* The small change in the number of observations in each subgroup is due to the exclusion of observation with $w^{-1} > 0.02$. This is done to exclude the spurious observations below 50 km/sec (see § II) *and* to avoid a possible singularity in w^{-1} (which can be seen by solving eq. [1] for w^{-1}).

† The last line in the table (for $w_t = 9$ km/sec) best represents the over-all sample of comet observations under discussion here.

of w_r can be compared with other evidence because there are no other values of w_t. The discussion by Snyder *et al.* (1963) and the discussion given here in § III (see Table 2) indicates that a mean representative value is $w_r \approx 500$ km/sec. Thus, our sample of comet observations is *deficient* in observations of high velocity and high ΣK_p. The inclusion of Comet 1942G raises the mean w_r from 380 km/sec to 448 km/sec: the latter value is fairly close to the figure of 500 km/sec believed to be representative. The inclusion of the observations of Comet 1942G substantially relieves the deficiency of observations with high ΣK_p, and apparently makes the sample of comet observations more nearly representative of typical interplanetary conditions. Thus, the inclusion of Comet 1942G in the determination of w_t appears to be correct on the basis of the data presently available.

The variation of w_t with ΣK_p implied in the last paragraph is qualitatively in the sense expected by theory and lends support for the values found. If conditions are quiet and undisturbed in the interplanetary medium, then one expects the plasma and the magnetic field to interact more or less according to the analogy suggested by Axford *et al.*

(1963). Here the field lines corotate with the Sun like the grooves in a phonograph record while the plasma moves through the field like the needle on a record. In this situation, there is little interaction between the field and the plasma, and little torque is exerted on the plasma by the corotating field. However, when the situation is disturbed by the penetration of an enhanced beam, the simple, smooth situation cannot exist, and the Sun may exert a torque on the enhanced plasma through the lines of force.[1] Thus, the situations expected and found are the same. It is noted that the tangential velocities found here are entirely consistent with the upper limits found by Antrack *et al.* (1962).

The tangential velocity at Earth has been estimated in the past by assuming solid-body rotation of the plasma with the Sun out to a cutoff distance, and conservation of angular momentum beyond. Axford *et al.* (1963) have estimated the extent of the region of solid-body rotation as 10 solar radii, a number which leads to a tangential velocity of $\sim$1 km/sec at the orbit of Earth. Within this simple framework, 10 km/sec tangential velocity at Earth implies corotation to about 30 solar radii. The larger magnetic fields associated with the active regions and thus presumably with the M region streams may be able to enforce corotation to substantial distances.

The tangential component of the plasma velocity is of interest in connection with the aberration angle of Earth's magnetic tail. For typical conditions, say, $w_r = 500$ km/sec, the angle ϵ_B is $3\overset{\circ}{.}4$ and $2\overset{\circ}{.}3$ for a w_t of zero and 10 km/sec; for quiet conditions, say, $w_r =$ 350 km/sec and $w_t = 5$ km/sec and zero, $\epsilon_B = 4\overset{\circ}{.}1$ and $4\overset{\circ}{.}9$, respectively; for disturbed conditions, say, $w_r = 750$ km/sec and $w_t = 15$ km/sec, $\epsilon_B = 1\overset{\circ}{.}1$. Observation of the position of the geomagnetic tail under varying conditions could provide a direct test of the picture presented here.

The tangential velocity may also be of interest in connection with the angular momentum of the Sun. The solar wind carries off angular momentum, and, as Dicke (1964) has pointed out, this situation could be effective in maintaining a regime in which the Sun has a rapidly rotating core but a slowly rotating atmosphere. This subject bears on the question of the dynamical eccentricity of the Sun, and, further, on the advance of the perihelion of Mercury as a test of general relativity. If the situation is idealized to angular momentum being carried through a cylinder of total height equal to 1 a.u. at the orbit of Earth by a solar wind with $w_r = 500$ km/sec, $N_e = 4/\text{cm}^3$, and $w_t = 10$ km/sec, the Sun loses angular momentum at the rate of $2\pi r_E{}^3 N_e m_H w_r w_t = 7 \times 10^{30}$ gm cm^2/sec^2. The total angular momentum of the Sun is about 1.7×10^{48} gm cm^2/sec (Allen 1963, p. 161). Thus, we have an *e*-folding time for solar rotation of 7×10^9 years, a time which is close to the presently accepted age of the Sun, 5×10^9 years (e.g., Schwarzschild 1958); the deceleration time could be somewhat lower ($\sim \times 4$) on the basis of the higher (but preliminary) densities found from Massachusetts Institute of Technology's IMP plasma experiment (Lyon 1965). Thus, the sink of angular momentum supplied by the corona and the solar wind may well be important in the structure and evolution of the Sun.

V. THE SAMPLE OF COMET OBSERVATIONS AND SOME POSSIBLE RESULTS OF THE BIAS

One may well now inquire about possible variations of the velocity of the solar wind as a function of other variables such as heliocentric distance, heliocentric latitude, and phase of the sunspot cycle. Pflug (1965) has reported certain results relating to these variables, and we return subsequently to a discussion of Pflug's results.

To show how our sample is distributed, we have constructed histograms by weight and by number of the observations as a function of distance, latitude, and phase in the solar cycle; direct and retrograde comets are plotted separately in Figures 5–7. The bias is immediately seen to be severe in all cases except possibly the distribution in heliocentric distance.

[1] I am indebted to Professor L. Biermann for correspondence concerning this point.

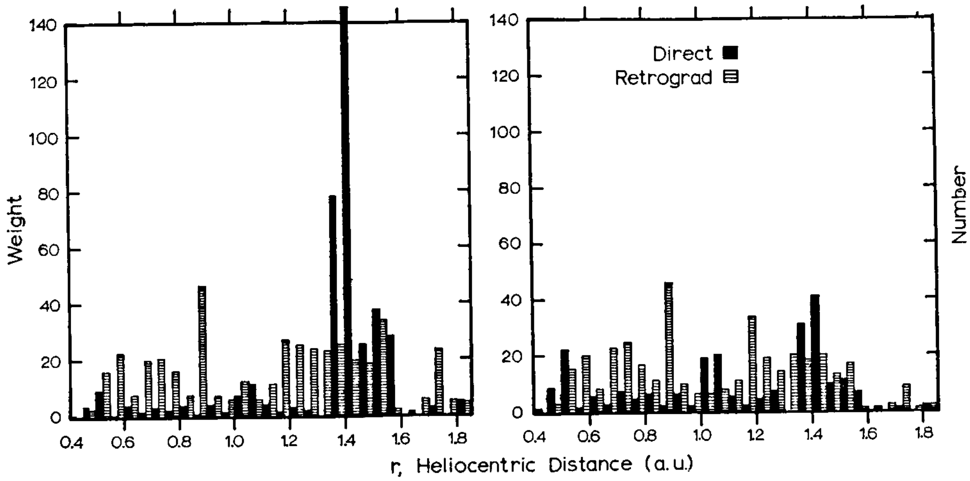

Fig. 5.—Histogram showing the distribution of direct and retrograde comets with heliocentric distance by weight and by number. Each interval of 0.05 is split into two sections with the left-hand side showing the data for direct comets and the right-hand side for retrograde comets. This graph shows the *best* distribution for a geometrical or temporal variable; see Figs. 6 and 7.

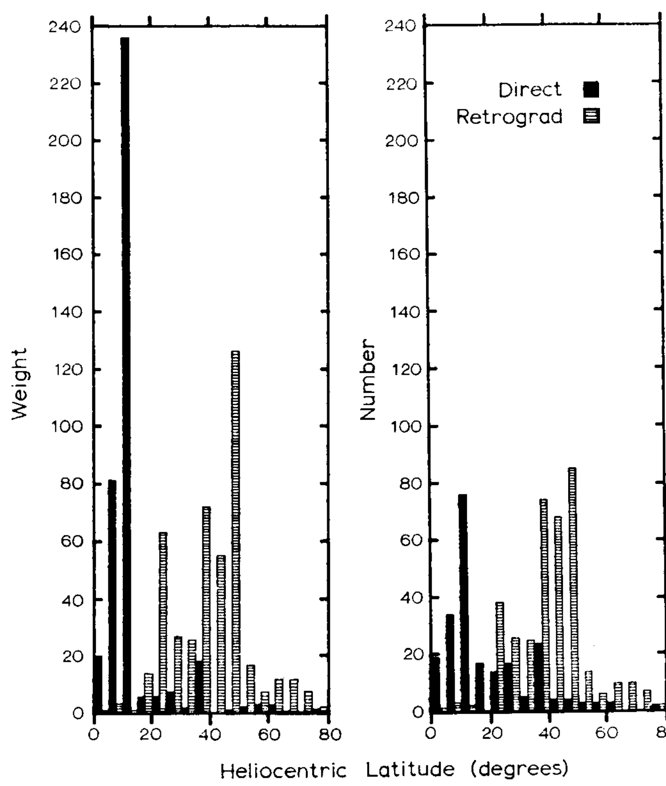

Fig. 6.—Histogram showing the distribution of direct and retrograde comets with heliocentric latitude by weight and by number. The format follows that of Fig. 5. Note that direct comets are preferentially *observed* at lower latitudes, while retrograde comets are preferentially *observed* at higher latitudes.

The velocity as a function of distance is easily investigated as is shown in Table 3. The data in the table indicate a sharp *increase* in the plasma velocity beyond Earth at a heliocentric distance of about 1.4 a.u. This result is clearly ridiculous in terms of a heliocentric variation and apparently is caused by one comet (1942G) with high weight observations being at a distance of approximately 1.5 a.u. during a time of relatively high geomagnetic and solar activity. Thus the variation seen is easily explicable in terms of the relation between w and ΣK_p as discussed in § III.

The latitude dependence of the solar wind found by Pflug (1965) is in the sense that the velocity is a minimum between the solar latitudes of 20°–30°; the same result is found from the sample of observations available in the comet catalogue. In terms of solar physics, this result is very difficult to understand. Statistically, it seems to result from the fact that the comet 1942G (with high velocities) lies at very low latitudes. Note that the result appears to stem from the *location of the comet* and the explanation says *nothing*

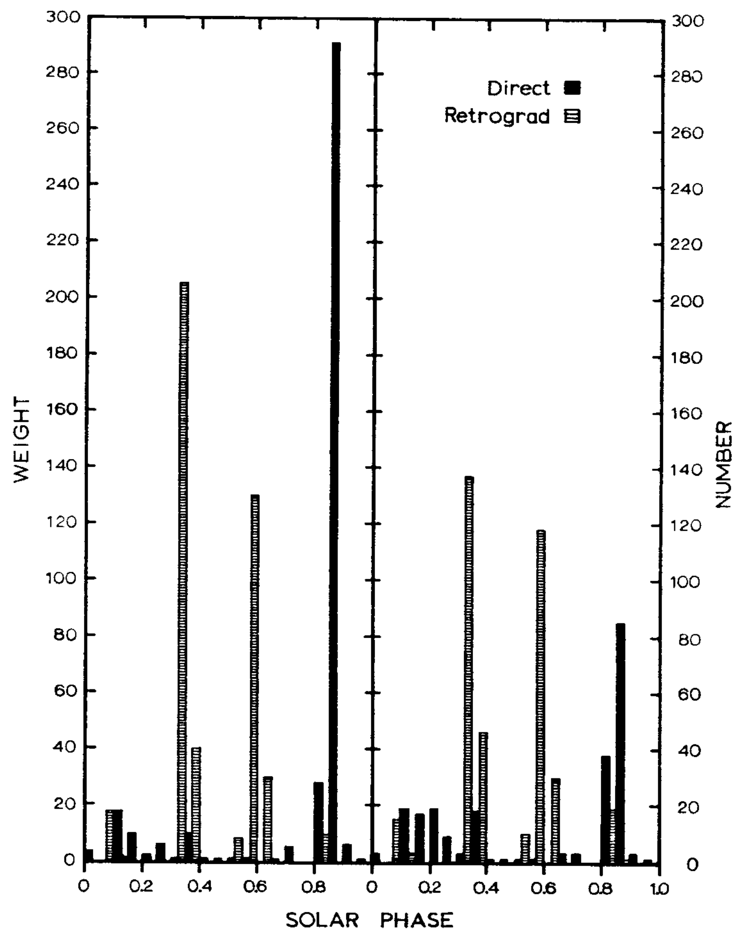

FIG. 7.—Histogram showing the distribution of direct and retrograde comets with solar phase by weight and by number. Solar phase here runs linearly from minimum to minimum. Hence, minimum is at phase 0.0 and maximum at phase 0.41 (on the average). The format follows that of Figs. 5 and 6. Besides the rather spotty distribution, note that on the average we *observe* retrograde comets at solar maximum and direct comets near minimum.

about the latitude distribution of plasma velocities in solar M regions. The current ideas (discussed above) concerning the angular extent of M region–associated beams ascribe sizes $\approx 45°$ and mean that it will be very difficult to establish any latitude dependence of the plasma velocity. The minimum requirement is that of identification of the sites of the origin of the plasma (e.g., the M regions) on the Sun, and, as we have noted, this cannot be done. Indeed, it appears that a dependence of the plasma velocity on latitude cannot be established with the data presently available.

Lastly, the comparison can be made between years of solar maximum and solar minimum. Since comet 1942G falls in years of "solar minimum," the analysis actually shows a plasma velocity greater at solar minimum than at solar maximum as previously reported (Brandt 1965). Thus, the straightforward approach of obtaining dependences for the solar-wind velocity in the biased sample is not adequate. It is necessary to identify a physical variable and calibrate it in the sample to obtain "meaningful" results. This is the approach (hopefully) adopted in § III where the relation between ΣK_p and w is discussed. All of the "pseudo-variations" of plasma velocity described here (with heliocentric distance, latitude, and solar cycle) are simply explicable on the basis of a biased distribution and the relation between w and ΣK_p.

The sample of comet observations available to Pflug and me must be rather similar, particularly since both samples depend strongly on the observations of Hoffmeister (1943). Thus, it seems that the curious latitude dependence found by Pflug (1965, 1966) should tentatively be ascribed to the result of bias in the sample of comet observations.

VI. DISCUSSION

Here we briefly review the results and discuss their relation to other investigations. The basic methods and philosophy were described in § I, and a lower bound of 150 ± 50 km/sec to the plasma velocity was derived in § II. This lower bound is entirely consistent with the results of § III as shown in Table 1; the "intrinsic dispersion" is roughly 200 km/sec, which leads to a lower bound compatible with the result directly determined.

The major result of § III confirms the dependence of the plasma velocity on geomagnetic activity as first found by Snyder *et al.* (1963), and as extended by Hirshberg (1965). It should be reiterated that the primary value lies in the independent confirmation of the relationship and its extension to a wider range of heliocentric distances, solar latitudes, and phases in the sunspot cycle; the numerical agreement found must be at least partially fortuitous. The results could be influenced, for example, if the motion of plasma associated with geomagnetic disturbances showed large systematic departures from the radial direction other than the simple, tangential velocity included here.

It should be noted that the Mariner II, comet, and storm data nicely compliment one another. The Mariner data are the most accurate, but severely limited in space and phase of the solar cycle. The comet observations are relatively inaccurate, but cover a wider area in space and, particularly, phase of the solar cycle. The storm data extended the relationship into the realm of high geomagnetic indices and higher velocities which are not covered by the Mariner II or comet data. With the present state of the art, the minimum velocity of the solar wind and the tangential velocity can be studied only through the comet data.

Evidence for a tangential component for the solar plasma velocity of about 10 km/sec (but varying with solar activity) was presented in § IV; possible bias in the sample, namely, the fact that retrograde comets tend to come at solar maximum (and vice versa) in our sample may cloud the issue somewhat (see Fig. 7). However, the agreement with a qualitative theoretical picture together with the absence of a significant variation in the mean value of the plasma velocity from solar maximum to minimum supports the reality of the results found. These may be of interest in discussions of Earth's magnetic tail and concerning the disposition of solar angular momentum.

The sample of comet observations was found to be biased in § V. This bias may be responsible for the curious latitude dependence of the solar plasma velocity found by Pflug (1965) where the plasma velocity is *least* in the sunspot zones (latitude 20°–35°). However, the general run of velocities, namely, 400 to 500 km/sec, found by Pflug (1965) agrees with the results found here, the Mariner II results (Snyder *et al.* 1963), the preliminary results from IMP-1 of 360 ± 40 km/sec (Lyon 1965), and the value of 385 ± 45 km/sec found by Ness and Wilcox (1964) from a sampling of the interplanetary magnetic field structure through an experiment on IMP-1 and by seeking a correlation with photospheric magnetic structure.[2]

This investigation concerns only Type I comets and gives no direct information concerning plasma densities. The heliocentric distance probed extends to approximately 1.8 a.u. which is generally the limit of th^ extent of Type I comets. Belton (1965*b*) has investigated the properties of Type II tails (including the ones associated with distant comets) and found that much of the evidence for a transition region in velocity near 2 a.u. as suggested by Brandt (1962) no longer exists. However, certain properties of comets and, perhaps, cosmic rays (specifically, their storage) require explanation. Perhaps the interplanetary plasma is not involved or perhaps a different property (other than velocity) undergoes a change.

Beyer (e.g., 1953) has presented considerable evidence which shows that the brightness of comet comas is closely correlated with sunspot number. A number of independent investigations have shown that the solar plasma velocity is not correlated with sunspot number, and, moreover, that the mean plasma velocity does not vary markedly over the solar cycle. Hence, the velocity of the solar plasma cannot be the property responsible for the brightness of comet comae. The sunspot number gives an indication of the number and extent of active regions on the Sun. Their primary characteristic (besides possible plasma emission) is an enhancement of extreme ultraviolet and X-ray emission. Thus, it is possible that energetic photons and not plasma are responsible for the production of the material which scatters solar radiation to produce the observed comet comae. This possibility is not surprising, but it serves once again to emphasize the independent physical nature of the coma and the tail; an alternate, but unlikely, interpretation would attribute changes in coma properties to changes in the plasma density and temperature.

The chief difficulty with this investigation concerns the large attrition of comet observations. Various observations must be thrown out because they are visual (Beyer 1947; Belton and Brandt 1966), of Type II comets, of comets with unseparated Type I and Type II tails, of comets observed only while Earth was nearly in the plane of the comet's orbit, of comets before 1932 (and hence no K_p values), and of comets with latitudes greater than a specified value. Thus the investigation began with about 1600 observations and, when a possible latitude dependence of the ΣK_p versus w relation was investigated in § III, only 100–300 observations remained. It is fortunate that the w versus ΣK_p relation is predominantly due to Comets Whipple-Fedtké-Tevzadze (1942G) and Mrkos (1957D) because both comets are well observed and because the large attrition precludes any statistical niceties. This situation underscores the need for additional reduction of older plates and the continuous patrol of new comets.

[2] Note, however, that the velocities found by Lüst (1961) for storms producing activity in comet tails (increased number and size of tail knots, etc.) are generally quite low compared to the value $\approx 10^3$ km/sec which should apply. Possibly the comparison requires the identification of the equivalent of the SSC event on Earth (Hirshberg 1965) whereas, very crudely, the increased activity may correspond to a period late in the main phase of a major storm. This could be the case if the comet is enveloped by the storm plasma, magnetic disturbances are propagated down to the nucleus, the tail plasma (CO^+) for the enhanced knots is produced, and then the enhanced knots exit from the coma where they are observed.

I am indebted to Dr. M. J. S. Belton for his interest throughout the course of this investigation; valuable conversations and correspondence with Professor L. Biermann are also acknowledged. Dr. Joan Hirshberg kindly supplied the points for the storm data. Portions of the calculations were performed by Mrs. Linda Scheer and Marvin W. Stephens.

Comments by anonymous referees have improved the presentation of this material.

APPENDIX

COMETS USED IN THIS INVESTIGATION, NUMBER OF OBSERVATIONS, AND THEIR TOTAL WEIGHT

Name	Designation	No.	Weight
Davidson	1889E	1	0.87
Swift	1892A	19	2.17
Rordame-Quenisset	1893A	9	4.80
Brooks	1893D	26	45.54
Gale	1894B	11	9.62
Perrine	1895C	2	1.41
Perrine-Lamp	1896A	1	0.99
Swift	1896B	1	0.94
Perrine	1898B	3	5.26
Swift	1899A	9	4.66
Perrine-Borrelly	1902B	17	17.84
Borrelly	1903C	32	19.42
Giacobini	1905C	7	0.12
Morehouse	1908C	148	159.87
Kiess	1911B	10	4.63
Brooks	1911C	38	28.05
Gale	1912A	1	1.22
Peltier	1936A	9	6.07
Finsler	1937F	44	37.12
Kozik-Peltier	1939A	2	1.28
Jurlof-Achmarof-Hassel	1939D	7	5.90
Cunningham	1940C	3	0.61
Whipple-Fedtké-Tevzadze	1942G	87	295.75
Rondanina-Bester	1947B	1	0.69
Honda-Bernasconi	1948G	1	0.58
Abell	1953G	3	3.46
Mrkos	1955E	1	1.53
Honda	1955G	1	0.87
Perrine-Mrkos	1955I	1	0.00
Olbers	1956A	2	6.74
Encke	1957C	2	0.06
Mrkos	1957D	104	158.20
Ikeya	1963A	1	0.12
		604	826.39

REFERENCES

Allen, C. W. 1944, *M.N.*, **104**, 13.
———. 1963, *Astrophysical Quantities* (2d ed.; London: Athlone Press).
Antrack, D., Biermann, L., and Lüst, Rh. 1964, *Ann. Rev. Astr. and Ap.*, **2**, 327.
Axford, W. I., Dessler, A. J., and Gottlieb, B. 1963, *Ap. J.*, **137**, 1268.
Bartels, J. 1932, *Terr. Mag. Atm. Elect.*, **37**, 1.
———. 1962, *IAGA Bulletin No. 18* (Amsterdam: North-Holland Publishing Co.).
———. 1963, *Ann. géophys.*, **19**, 1.

Belton, M. J. S. 1965*a*, Paper presented at the Liège Astrophysical Symposium on "The Nature and Origin of Comets," July.
———. 1965*b*, *A.J.*, **70**, 451.
Belton, M. J. S., and Brandt, J. C. 1966, *Ap. J. Suppl.*, **13**, 125 (No. 117).
Beyer, M. 1947, *Astr. Nachr.*, **275**, 237.
———. 1953, in *La Physique des Comètes* (Louvain: Ceuterick), p. 236.
Biermann, L. 1951, *Z. f. Ap.*, **29**, 274.
———. 1953 in *La Physique des Comètes* (Louvain: Ceuterick), p. 251.
Biermann, L., and Lüst, Rh. 1964, in *The Moon, Meteorites, and Comets*, ed. B. M. Middlehurst and G. P. Kuiper (Chicago: University of Chicago Press), p. 618.
Brandt, J. C. 1961, *Ap. J.*, **133**, 1091.
———. 1962, *Icarus*, **1**, 1.
———. 1965, Paper presented at the Liège Astrophysical Symposium on "The Nature and Origin of Comets," July.
Brandt, J. C., and Cassinelli, J. R. 1966, *Icarus*, **5**, 47.
Chamberlain, J. W. 1961, *Physics of the Aurora and Airglow* (New York: Academic Press), p. 109.
Curtiss, R. H. 1903, *Lick Obs. Bull.*, **2**, 99.
Dicke, R. H. 1964, *Nature*, **202**, 432.
Hirshberg, J. 1965, *J. Geophys. Res.*, **70**, 4159.
Hoffmeister, C. 1943, *Z. f. Ap.*, **22**, 265.
Lüst, Rh. 1961, *Z. f. Ap.*, **51**, 163.
Lyon, E. F. 1965, private communication.
Maer, K., and Dessler, A. J. 1964, *J. Geophys. Res.*, **69**, 2846.
Mammano, A., and Wurm, K. 1965, *Icarus*, **4**, 1.
Mustel, E. 1964, *Space Sci. Rev.*, **3**, 139.
———. 1965, *Soviet Astr.—A.J.*, **9**, 375.
Ness, N. F., and Wilcox, J. M. 1964, *Phys. Rev. Letters*, **13**, 461.
Parker, E. N. 1958, *Ap. J.*, **128**, 664.
Pecker, J-C., and Roberts, W. O. 1955, *J. Geophys. Res.*, **60**, 33.
Pflug, K. 1965, Paper presented at the Liège Astrophysical Symposium on "The Nature and Origin of Comets," July.
———. 1966, *Pub. Ap. Obs. Potsdam*, No. 106.
Priester, W., and Cattani, D. 1962, *J. Atm. Sci.*, **19**, 121.
Saemundsson, Th. 1962, *M.N.*, **123**, 299.
Saito, T. 1965, *J. Geomag. and Geoelect.*, **17**, 23.
Schwarzschild, M. 1958, *Structure and Evolution of the Stars* (Princeton, N.J.: Princeton University Press).
Snyder, C. W., Neugebauer, M., and Rao, U. R. 1963, *J. Geophys. Res.*, **68**, 6361.